Complicated dynamics in a reversible Hamiltonian system near a symmetric heteroclinic contour

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Hamiltonian systems arise as mathematical models in many branches of physics, chemistry, engineering. Such systems as their study shows have usually a rather complicated structure that leads to great difficulties in their examination. Therefore one of the fruitful method of their investigation is the study of the orbit behavior near some specific structures which can be distinguished by simple conditions. The study of a system near a homoclinic orbits or contours made up of several heteroclinic orbits and equilibria or periodic orbits is undoubtedly one of such problem.

We study the dynamics of an analytic reversible Hamiltonian system X_H with two degrees of freedom assuming the system has a heteroclinic contour involving a symmetric saddle-center equilibrium p (its eigenvalues are nonzero numbers $\pm i\omega, \pm\lambda, \omega, \lambda \in \mathbf{R}$), an orientable symmetric saddle periodic orbit γ lying in the same level of Hamiltonian $H = H(p)$ and two nonsymmetric heteroclinic orbits Γ_1, Γ_2 joining p with γ and interchanged by the involution L, $\Gamma_2 = L(\Gamma_1)$. The reversible involution L is supposed to have a smooth two-dimensional set $Fix(L)$ of its fixed points. Such a system are met in generic one-parameter families of reversible Hamiltonian systems.

Saddle periodic orbit γ belongs to a 1-parameter family γ_c of saddle periodic orbits in all close levels $H = c$ forming a symplectic cylinder. Reversible Hamiltonian systems possessing the above mentioned contour can be of two different types in dependence on how the involution acts locally near a saddle-center.

Our results demonstrate the existence in such a system:

- countable set of transverse 1-round homoclinic orbits to γ and related to them non-uniformly hyperbolic subsets;

- appearance for $c > 0$ of two transverse heteroclinic contours involving γ_c, a small Lyapunov periodic orbit l_c near p and four heteroclinic orbits $\Gamma_1^\pm$ and $\Gamma_2^\pm = L(\Gamma_1^\pm)$

and related with them uniform hyperbolic subsets;

- a finite set of transverse 1-round homoclinic orbits to γ_c for $\mid c \mid$ close to $H(p)$ and uniformly hyperbolic sets related with them;

- a countable set of values $c_n < 0$ accumulating at $c = 0$ such that on the level $H = c_n$ the system has a tangent homoclinic orbit to γ_{c_n} and bifurcations nearby orbits related to this tangency;

- countable sets of saddle and elliptic periodic orbits.

Some other bifurcation phenomena will be discussed when generic one parameter reversible unfoldings of such a system are considered.