Taylor dispersion and Turing-like instabilities of flames

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Summary We investigate the effect of Taylor dispersion on the thermo-diffusive instability of flames. This is a physically interesting and analytically tractable problem within a relatively unexplored class of problems pertaining to the interaction between Taylor dispersion (or flow-enhanced diffusion) and Turing-like instabilities in reaction-diffusion systems. The analysis is carried out in the Hele-Shaw burner configuration and adopts a constant density approximation. Depth-averaged equations are first obtained which incorporate Taylor dispersion and show that diffusion is effectively anisotropic. A linear stability analysis of travelling wave solutions leads to a simple dispersion relation which allows stability-bifurcation diagrams to be drawn in the parameters space whose physical implications are discussed.

INTRODUCTION

There are two major intrinsic flame instabilities in premixed combustion. The first, known as the Darrieus-Landau or hydrodynamic instability, is the instability of an interface propagating towards a less dense medium [1]. The second instability, known as the thermo-diffusive instability [2], occurs in fuel-lean mixtures where the thermal diffusivity D_T and the fuel diffusion coefficient D_F are such that their ratio, the Lewis number $\boxed{\mathrm{Le} \equiv D_T/D_F}$, is sufficiently away from unity. The thermo-diffusive instability may be identified as a Turing-like instability [3] since it requires, as in the case of Turing instability in a reaction-diffusion system [4], two diffusive processes to have differing diffusion coefficients. Strictly speaking, however, the Turing and thermo-diffusive instabilities are quite different, not least because the former pertains to a spatially homogeneous steady state, while the latter to a travelling wave.

The rich topic of flame instabilities has been the subject of dedicated reviews e.g. [3, 5, 6]. A main objective of this paper is to complement the literature by addressing unexplored aspects of flame instability using an extension of our approach based on the *thick flame asymptotic limit* described in [7, 8]. The specific novel problem to be addressed in this paper is ***the influence of Taylor dispersion on the thermo-diffusive instability***, which is an interesting subproblem of the more general problem of ***flame instabilities in a Hele-Shaw cell under forced convection***. In fact, the Hele-Shaw configuration is ideal for investigating the instabilities of flames [9], at least in theoretical and numerical studies where heat-losses and density variations, which are difficult to ignore in practice, may be switched off or on, depending on our focus. Recent experiments on flames in Hele-Shaw cells have been conducted at the University of Aix-Marseille [10] and at the University of Southern California [11].

THEORETICAL RESULTS

As a configuration for the investigation we shall adopt that of the Marseilles experiment which is sketched in Figure 1 showing a flame propagating downwards. As described in [10], the burner's operation is briefly as follows. By opening the inlet valve at the bottom, a reactive mixture flows upwards to fill the cell and is ignited at the top of the burner where a flame parallel to the horizontal x-direction is formed and remains anchored thanks to a flow velocity exceeding on average the flame speed U_L. Closing the valve, the flow is stopped and the downwards flame propagation is recorded. It is important to point out that the effect of the flow, or forced convection, on the propagation and stability of the flame has not been addressed. Although this seems to be a simple ingredient to incorporate, it has a profound influence. Specifically, by considering flame propagation against a vertical flow, say given in the unburnt-gas by $\mathbf{u} = u_0(1 - y^2/a^2)\hat{\mathbf{z}}$ where a is the channel half-width and $\hat{\mathbf{z}}$ a unit vector in the z-direction, it is possible to investigate theoretically the effect of Taylor dispersion [12, 13] on flame propagation and stability. Indeed, Taylor dispersion modifies the effective diffusion coefficients of heat and mass in the flow direction (z-direction), which modifies the effective Lewis number $\mathrm{Le}_{\mathrm{eff}}$. In fact, the results of our publication [8] indicate that $\boxed{\dfrac{\mathrm{Le}_{\mathrm{eff}}}{\mathrm{Le}} = \dfrac{1 + \gamma(1-\alpha)^2\mathrm{Pe}^2}{1 + \gamma(1-\alpha)^2\mathrm{Pe}^2\mathrm{Le}^2}}$, where $\mathrm{Pe} = au_0/D_T$ is the Peclet number, $\gamma = 8/945$ a numerical coefficient, and α the gas expansion parameter defined in terms of the unburnt gas and burnt gas densities ρ_u and ρ_b by $\alpha = 1 - \rho_b/\rho_u$. The formula shows that $\mathrm{Le}_{\mathrm{eff}} \sim \mathrm{Le}$ as $\mathrm{Pe} \to 0$ and $\mathrm{Le}_{\mathrm{eff}} \sim \mathrm{Le}^{-1}$ as $\mathrm{Pe} \to \infty$. This suggests that Taylor dispersion may have significant influence, worth exploring, on the thermo-diffusive instability which critically depends on $\mathrm{Le}_{\mathrm{eff}}$. This is a highly original investigation which is best initiated by adopting first

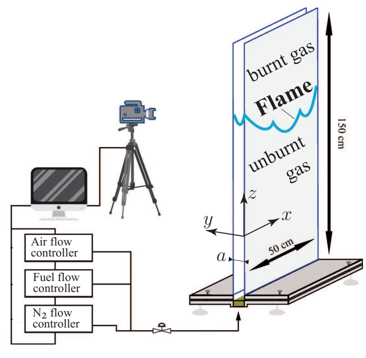

Figure 1: Hele-Shaw burner, adapted from [10]

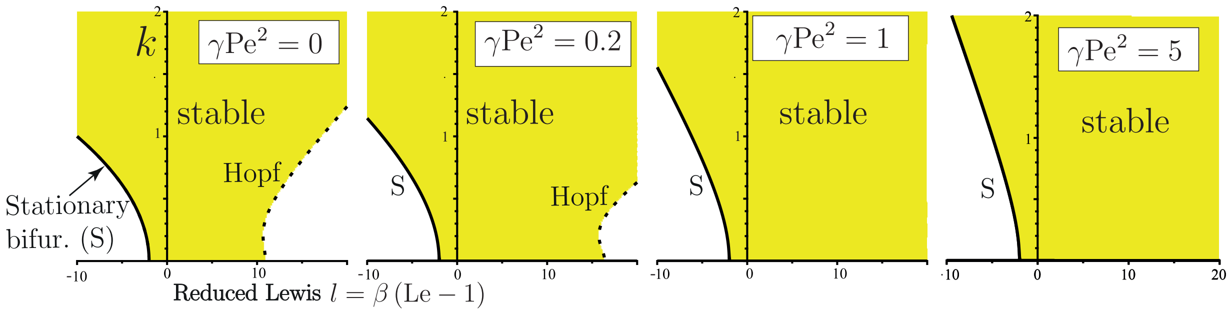

Figure 2: Stability region and bifurcations in the l-k plane for selected values of the parameter $\gamma\mathrm{Pe}^2$; l is the reduced Lewis number and k is the perturbation wave number.

the constant density approximation ($\alpha = 0$) and addressing the stability of a planar flame (parallel to the x-axis) propagating downwards against the flow. The problem has however a crucial complication: enhanced diffusion, which leads to the formula for $\mathrm{Le}_{\mathrm{eff}}$ above, is applicable in the longitudinal z-direction; in the transverse x-direction diffusion is unaffected by the flow. Therefore, the stability problem in the x-z plane (obtained by y-averaging the governing equations), is one involving ***anisotropic diffusion***. Accordingly, the classical approach of tackling the problem, using familiar jump conditions applicable at inner reaction layers needs significant revision. This revision and related aspects are addressed in this work. The main outcome is the derivation of a dispersion relation describing the linear stability of the flame to perturbations proportional to $\exp\,(ikx+st)$ with wave number k and growth rate s. The dispersion relation is found to be given by

$$2\Gamma^2(\Gamma-1) + \frac{l}{1+\gamma\mathrm{Pe}^2}\left[(\Gamma-1-2s)(1-\gamma\mathrm{Pe}^2) + 4\gamma\mathrm{Pe}^2k^2\right] = 0\,,$$

where $l = \beta(\mathrm{Le}-1)$ is the so-called reduced Lewis number involving the Zeldovich number β and $\Gamma = \sqrt{1+4s+4k^2}$.
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CONCLUDING REMARKS

We note that the dispersion relation incoroprates the effect of Taylor dispersion and the resulting anisotropy of diffusion. A sample of the results implied by the dispersion relation are illustrated in figure 2 for selected values of the parameter $\gamma\mathrm{Pe}^2$. When $\gamma\mathrm{Pe}^2 = 0$ we recover the classical results with a stationary bifurcation curve for small values of l and a Hopf bifurcation curve for larger values. As $\gamma\mathrm{Pe}^2$ is increased the Hopf bifurcation curve is displaced to the right and disappears for $\gamma\mathrm{Pe}^2 > 1$. Thus a sufficiently strong flow impedes the oscillatory flame instability. Also, the size of the stability domain in the left half-plane (delimited from the left by the stationary bifurcation curve) is reduced by an increase in $\gamma\mathrm{Pe}^2$ which indicates that the flow may somewhat promote the cellular instability. In any case, the results demonstrate the ability of Taylor dispersion to significantly affect Turing-like instabilities.

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