Local Lyapunov Exponent in the Standard Map

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ABSTRACT

We studied the manifestation of chaos in a Hamiltonian system which is the standard map, to see the phase space trajectories evolution and the Local Lyapunov Exponent. The dynamics of the Hamiltonian system under consideration is described by the discrete equations of the standard map.

$$\begin{cases} p_{n+1} = p_n + k \sin\theta_n \\ \theta_{n+1} = \theta_n + p_{n+1} \end{cases} \quad (1)$$

Where k is the perturbation parameter**,** n is the iterations number. However, time t is measured in the period T unity, so t = n is a continuous variable.

We use Local Lyapunov Exponent (LLE) because numerically only finite and limits quantities are possible to calculate.

To follow the separation progress between two trajectories reverse time in the phase space, we calculate the LLE of these two trajectories. For this, we consider two trajectories $x_1(0) = (q_1(0), p_1(0)) = (q1_0, p1_0)$ and $x_2(0) = (q_2(0), p_2(0)) = (q2_0, p2_0)$ with initial separation $d_0 = (x_2 - x_1)$.

We define Local Lyapunov Exponent (LLE) as:

$$L(x(0), d_0, t) \equiv \frac{1}{t} \ln\left[\frac{d(t)}{d_0}\right] ; \quad \text{when : } d_0 \to 0, t \to \infty \qquad (2)$$

Which $L(x(0), d_0, t)$ depends on the initial separation distance between two trajectories $d_0 = |d_0|$ and time t. The displacement in phase space at time test is $d(t) = |d(t)|$. And the double limit of LLE when d_0 tends to zero and t tends to infinity is defined as the Global Lyapunov Exponent (GLE) λ, which is not, in general, not dependent on the initial position in the phase space.

In the non-chaotic regime k = 0.96, the separation distance between two adjacent trajectories does not change over time, hence the superposition of the two trajectories on one another, depending on the time, which explains the regularity of its phase space. While in the chaotic regime k = 10, the separation distance between two trajectories believe exponentially and LLE is constant in the interval of small time for different initial separation distances, and then it decreases asymptotically and quickly to approach zero for large time, which explains the appearance of the chaotic sea that fills all the phase space.

Generally, in a chaotic regime, for any initial separation distance between two trajectories approaching zero, the Local Lyapunov Exponent corresponding decreases asymptotically as its upper limit function $O\left(\frac{Ln\ N}{N}\right)$.