From isotropic turbulence to transition in plane channel flow

Wouter Bos *[1] and Faouzi Laadhari[1]
[1] *CNRS, LMFA, Ecole Centrale de Lyon, UCBL, INSA Lyon, Univ Lyon*

Summary In plane Poiseuille flow, somewhat beyond a critical value of the Reynolds number, laminar flow co-exists with turbulent stripes. We transpose modeling concepts, which were derived for the case of statistically isotropic turbulence, to this case. Thereby it is shown that the kinetic energy balance around the onset of turbulent flow in plane channel flow can be described by a simple model-equation. We first validate our ideas in the isotropic setting using two-point closure technique. Subsequently we assess the ideas applied to channel flow using high resolution direct numerical simulations.

INTRODUCTION

Transitional shear flows and the modeling of isotropic turbulence are two seemingly different research subjects. Recently, however, it was realized that isotropic turbulence and the laminar-turbulent transition in wall-bounded shear flows share certain features [1]. For instance, it was shown that relaminarization, a feature generally associated with shear flows such as flow through pipes driven by a pressure gradient, can also be observed in statistically isotropic periodic box turbulence. In the present investigation we first investigate the transitional dynamics of isotropic turbulence. In particular, we will show that in the case of linear forcing a critical Reynolds number exists above which a turbulent flow can be sustained.

ISOTROPIC TURBULENCE

The global kinetic energy of a closed or periodic turbulent flow is fully determined by the production (p) and dissipation (ϵ) mechanisms. Its volume-averaged balance is simply given by the equation

$$\frac{dk}{dt} = p - \epsilon. \tag{1}$$

We consider the Navier-Stokes equations to which we add a linear forcing term [2],

$$\frac{D\boldsymbol{u}}{Dt} = \Pi + \nu\Delta\boldsymbol{u} + \alpha\boldsymbol{u}. \tag{2}$$

with $\boldsymbol{u}$ the velocity, $-\Pi$ the density-normalized pressure gradient and ν the kinematic viscosity. The parameter α determines the strength of the linear forcing. This system allows the exact determination of the energy injection as a function of the kinetic energy. We propose two models for the dissipation as a function of the kinetic energy and show that the analytical solutions to the modeled equation describe accurately the Reynolds number dependence of the kinetic energy of the flow obtained by triadic closure simulations [3]. Results of this study are shown in figure 1.

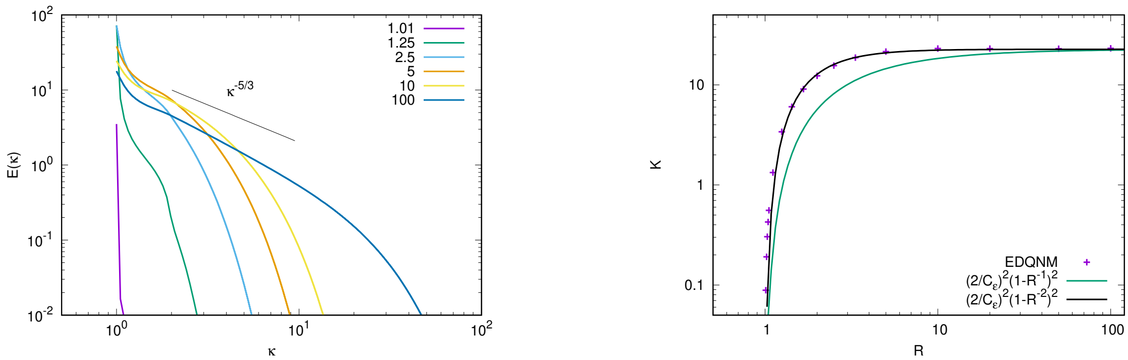

Figure 1: Closure analysis of the linearly forced Navier-Stokes equations allow to assess a simple model for the kinetic energy in steady states above a critical value of the Reynolds number. Left: energy spectra for forcing-rate-based Reynolds numbers $R = \alpha L^2/\nu = [1.01 : 100]$. Right: kinetic energy as a function of R. Two distinct models of the dissipation model are tested, leading to different analytical expressions for the kinetic energy. Symbols denote closure results, lines the model predictions.

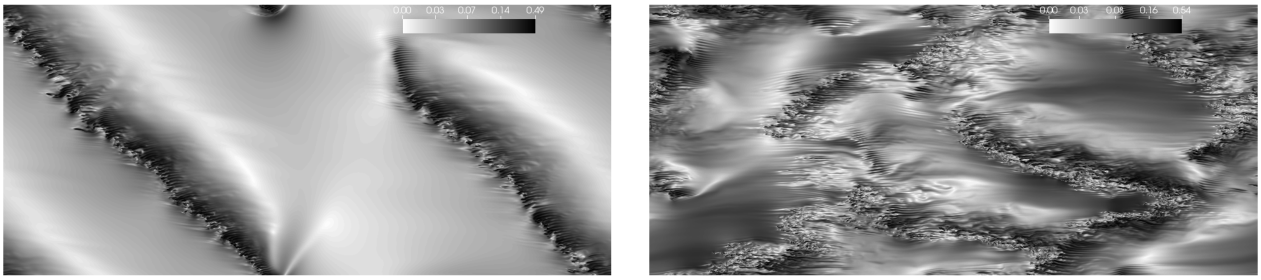

Figure 2: Left: DNS results of velocity-fluctuations in the center-plane of plane Poiseuille flow. The mean flow is directed from left to right. Right: kinetic energy as a function of the Reynolds number $R_\tau = \Gamma^{1/2}h^{3/2}/\nu$. Two distinct models (lines) are compared to the data.

PLANE POISEUILLE FLOW

Subsequently we transpose the ideas to the case of plane Poiseuille flow, i.e. flow between two parallel walls driven by a mean pressure gradient. In this case the evolution of the system is also given by equations (1) and (2), but the forcing of the system is now not determined by the linear forcing term ($\alpha = 0$) but by the imposed pressure gradient which we will call $-\Gamma$. Using the same type of model as for the isotropic case, we can obtain a closed expression for the kinetic energy as a function of the Reynolds number. We checked our ideas by means of a series of 25 pseudo-spectral simulations in a domain of size $L_x \times L_y \times L_z = 350h \times 2h \times 155h$ with periodic boundary conditions in the streamwise (x), and spanwise (y) directions. The domain is discretized using approximately 7×10^8 grid-points. Steady state values were obtained by decreasing stepwisely the Reynolds number starting from a turbulent initial condition.

In Figure 2, we show the RMS velocity fluctuations in the centerplane between the two walls for a Reynolds number close to the critical value below which the flow relaminarizes and a Reynolds number somewhat larger. Coexistence of laminar and turbulent regions is observed in the flow, in particular for the lowest value of the Reynolds number. Preliminary results, which will be consolidated and presented during the conference, show that our simple model for the kinetic energy as a function of the Reynolds number shows good agreement with the results of our simulations of channel flow.

CONCLUSIONS

It is shown that the behaviour of the turbulent kinetic energy in both isotropic turbulence and plane Poiseuille flow, near the critical Reynolds number, above which turbulence can be sustained, can be described by a simple model equation. What is still to be explored is how the results depend exactly on the size of the domain, and how the shape of the turbulent production term can be derived from the Navier-Stokes equations. Furthermore, in future work we want to assess our ideas in different systems which show subcritical transition, such as plane Couette flow.

References

[1] MF Linkmann and A Morozov. Sudden relaminarization and lifetimes in forced isotropic turbulence. *Physical review letters*, 115(13):134502, 2015.

[2] TS Lundgren. Linearly forced isotropic turbulence. *Annual Research Briefs-2003*, page 461, 2003.

[3] WJT Bos and F Laadhari. *A spectral analysis of the onset of linearly forced isotropic turbulence*. Submitted.

*Corresponding author. E-mail: wouter.bos@ec-lyon.fr.