A rotational Cauchy-Poisson problem

Peder Tyvand*[1,2]

[1]Peder Albert Tyvand (NMBU) – Faculty of Science and Technology P.O.Box 5003 NMBU, NO - 1432 Aas, Norway

[2]Norwegian University of Life Sciences – 1432 Aas, Norway

Abstract

The theory of water waves focusses on irrotational flows. We will design an unconventional water waves problem with an initial flow that is purely rotational, without any initial potential flow. We will investigate how the flow evolves in time due to free-surface nonlinearity in combination with conservation of vorticity for inviscid flow. A horizontal fluid layer has initially uniform thickness H. A surface elevation eta(x; t) will evolve, starting with eta(x; 0) = 0. An initial 2D normal mode with wavenumber k for the streamfunction psi (x; z; t) is:
psi(x; z; 0) = A(0) sin(kx) cos (pi z/(2H))

chosen to give vertical initial flow at the initial free surface z = 0, with an impermeable bottom at z = -H. A small-time expansion will be devloped for the coupled nonlinear problem where three nonlinear mechanisms are combined: (i) The nonlinear dynamic condition at the free surface. (ii) The nonlinear kinematic condition at the free surface. (iii) The nonliear vorticity equation. The solution for the streamfunction will be developed as a normal-mode Fourier series, combined with potential-flow contributions due to free-surface nonlinearity. We will discuss how higher Fourier components in the x and z directions will emerge from the coupled nonlinear effects at the surface and in the fluid by vortex interactions. These results are limited to 2D flows, but we will outline how this theory can be extended to 3D.

Keywords: free, surface flow, normal modes, vorticity equation

*Speaker