# Complicated dynamics in a reversible Hamiltonian system

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**Abstract**

Hamiltonian systems arise as mathematical mo dels in many branches of physics, chemistry, engineering. Such systems as their study shows have usually a rather complicated structure that leads to great difficulties in their examination. Therefore one of the fruitful metho d of their investigation is the study of the orbit behavior near some specific structures which can b e distinguished by simple conditions. The study of a system near
a homo clinic orbits or contours made up of several hetero clinic orbits and equilibria or p erio dic orbits is undoubtedly one of such problem. We study the dynamics of an analytic reversible Hamiltonian system $XH$ with two
degrees of freedom assuming the system has a heteroclinic contour involving a symmetric saddle-center equilibrium $p$ (its eigenvalues are nonzero numbers $\pm i\omega$, $\pm\lambda$, $\omega$, $\lambda \in \mathbf{R}$), an orientable symmetric saddle periodic orbit $\gamma$ lying in the same level of Hamiltonian $H = H(p)$ and two nonsymmetric heteroclinic orbits $1$, $2$ joining $p$ with $\gamma$ and interchanged by the involution $L$, $2 = L(1)$. The reversible involution $L$ is supposed to have a
smooth two-dimensional set $Fix(L)$ of its fixed points. Such a system are met in generic one-parameter families of reversible Hamiltonian systems. Saddle periodic orbit $\gamma$ belongs to a 1-parameter family $\gamma c$ of saddle periodic orbits in all close levels $H = c$ forming a symplectic cylinder. Reversible Hamiltonian systems possessing the above mentioned contour can be of two different types in dependence on how the involution acts locally near a saddle-center. Our results demonstrate the existence in such a system:
• countable set of transverse 1-round homoclinic orbits to $\gamma$ and related to them non-uniformly hyperbolic subsets;
• appearance for $c > 0$ of two transverse heteroclinic contours involving $\gamma c$, a small Lyapunov periodic orbit $lc$ near $p$ and four heteroclinic orbits $\pm 1$ and $\pm 2 = L(\pm 1)$
and related with them uniform hyperbolic subsets;
• a finite set of transverse 1-round homoclinic orbits to $\gamma c$ for — $c$ — close to $H(p)$ and uniformly hyperbolic sets related with them;
• a countable set of values $c\_n < 0$ accumulating at $c = 0$ such that on the level $H = c\_n$ the system has a tangent homoclinic orbit to $\gamma cn$ and bifurcations nearby orbits related to this tangency;
• countable sets of saddle and elliptic periodic orbits.
Some other bifurcation phenomena will be discussed when generic one parameter reversible unfoldings of such a system are considered.
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