Analysis of bursting dynamics in a modified facilitation-depression model accounting for afterhyperpolarization

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Abstract

Neuronal networks show periods of synchronous high-frequency activity called bursts separated by quiet time periods (interbursts). Such activation patterns can play an important role in the communication between neurons as well as for rhythm generation in the brain.

To study the bursting dynamics of a network, we develop a new stochastic model based on synaptic short-term facilitation and depression that also accounts for afterhyperpolarization, which shapes the end of a burst. A stochastic perturbation of this system leads to a succession of bursts and interbursts and we characterize their durations using a three dimensional phase-space analysis and numerical simulations. The phase-space contains three critical points (one attractor and two saddles) separated by a two-dimensional stable manifold (Sigma). Bursting is defined by long deterministic excursions outside the basin of attraction, while the interburst duration is defined by the escape induced by random fluctuations. To characterize the distribution of the burst durations, we determine the distribution of exit points located on the two-dimensional separatrix (Sigma) using WKB approximation and the method of characteristics to numerically solve the Fokker-Planck equation satisfied by this distribution. Finally, to study the interburst we account for the anisotropy of the dynamics around the attractor to reduce our analysis to a two-dimensional projection of the model. We analyze the two-dimensional phase-space by computing the probability density function of the trajectories in the phase-space which depends on the noise amplitude and satisfies the Fokker-planck equation.

Keywords: nonlinear dynamical systems, stochastic differential equations, Fokker, Planck equation, stochastic simulations, computational neuroscience

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