Birth of discrete Lorenz attractors in global bifurcations

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Abstract

Discrete Lorenz attractors are chaotic attractors, which are the discrete-time analogues of the well-known continuous-time Lorenz attractors. They are genuine strange attractors, i.e. they do not contain simpler regular attractors such as stable equilibria, periodic orbits etc. In addition, this property is preserved under small perturbations. Thus, Lorenz attractors, discrete and continuous, represent the so-called robust chaos.
Discrete Lorenz attractors can be observed in applications, but, until recent, there existed no tools to prove rigorously the presence of such attractors in particular models. In this talk, I present a series of results in this direction.

The first one is an analytic proof of existence of the Lorenz attractor in the classical Lorenz model. The latter appears as a rescaled normal form of bifurcations of triply degenerate equilibria and periodic orbits, so the appearance of such attractors is typical for these cases.

This result is used for the study of a certain class of maps (3D Henon maps) – when they possess a triply degenerate fixed or periodic point, then locally the map can be represented as a stroboscopic map of a periodically perturbed Lorenz-like system. This implies the existence of a discrete Lorenz attractor in the original 3D Henon map.

Finally, bifurcations of homoclinic and heteroclinic cycles are studied. It is shown that the first return map along such a cycle can be represented as a 3D Henon map after an appropriate change of coordinates. The main result proved here is the birth of infinitely many discrete Lorenz attractors in such bifurcations.
These results compose a big step towards a creation of a toolkit that will allow to prove rigorously the existence of a chaotic dynamics in models of Mathematics, Physics, Chemistry, Biology and others.

Keywords: Lorenz attractor, chaos, bifurcation, normal form

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