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# Anomalous nonlinear dynamics of media with complex structures

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## Abstract

This presentation is a brief review of studies of dynamic processes in structured media. Ideal crystalline structures are rarely met in material samples of a macroscopic size. Metals, concretes, and rocks commonly contain structural disruptions of different scales (dislocations, microcracks, ruptures) and/or consist of contacting structural elements (we call them grains in a general sense). In spite of such a variety, these materials often have a common feature: they consist of hard elements divided by relatively soft inclusions (bonds) such as grain contacts and cracks. As a result, the external force (stress) can create a strong deformation (strain) in the areas of bonds which causes a strong nonlinear response. The above features cause the following two properties which were observed, separately or, more often, together, in many experiments: 1. Hysteresis in the stress-strain dependence, and 2. Long-time relaxation (slow dynamics).

Although the bulk of experimental data has been accumulated since 1990s, the physical mechanisms underlying the above properties are not completely understood. Recently the authors suggested a unified model of nonlinearity in structured media which is based only on a few assumptions: the integral effect of contact ruptures with subsequent restoration - for the stress-strain hysteresis, and the presence of a low potential barrier which can be surmounted by thermal motion - for description of slow dynamics. The model naturally includes the classical, non-hysteretic nonlinearity for moderate loads and explains a commonly observed logarithmic law of relaxation, as well as deviations from that law.

Besides the physical model of nonlinear dynamics in the complex media, the wave propagation in such media is also studied. Some unsolved problems are outlined.

**Keywords:** Nonlinear dynamic elasticity, Hysteresis, Long, time relaxation, Waves in hysteretic media.

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